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EXERCISES

4.1 If the microstates of a stochastic network can be enumerated, one can solve the underlying dCME directly. For a network with m molecular species with r reactions, assume each molecular species can have at most n copies of molecules.

a) Without knowing the details of the reactions if one ignores all dependency between molecules and allow the possibility that all molecular species may simultaneously have the maximum of n copies of molecules. Provide an upper bound on the size of the state space.

b) As different molecular species are coupled through chemical reactions, they are not independent. Because of these couplings, the effective number of independent species is less than m. Let the stoichiometry matrix of the