Global Stability of Banking Networks Against Financial Contagion: Measures, Evaluations and Implications

Bhaskar DasGupta

Department of Computer Science
University of Illinois at Chicago
Chicago, IL 60607
bdasgup@uic.edu

September 16, 2014

- Based on the thesis work of my PhD student Lakshmi Kaligounder
- Joint works with Piotr Berman, Lakshmi Kaligounder and Marek Karpinski

UIC

Outline of talk

1. **Introduction**

2. **Global stability of financial system**
   - Theoretical (computational complexity and algorithmic) results
   - Empirical results (with some theoretical justifications)
   - Economic policy implications

3. **Future research**

UIC

Typical functions of financial systems in market-based economy

- ***borrowing*** **from surplus units**
- ***lending*** **to deficit units**

Financial stability (informally)

ability of financial system perform its key functions even in "stressful" situations

Threats on stability may severely affect the functioning of the entire economy

study of financial stability — some historical perspectives

research works during "Great Depression" era

- **Irving Fisher (1933)**
- **John Keynes (1936)**

Hyman Minsky (1977)

instabilities are inherent (*i.e.*, "systemic") in many capitalist economies

1930s great depression ⇒ **early 1980s recession** ⇒ **2007 recession**

1930s great depression:

- stock market collapse (black Tuesday)
- major bank failures
- high unemployment

2007 recession:

- stock market collapse
- real estate collapse
- major bank almost failures (averted with government aid)
- high unemployment

Why financial systems exhibit instability ?

- inherent property of system (*i.e.*, systemic) ?
- caused by “a few” banks that are “too big to fail” ?
- due to government regulation or de-regulation ?
- random event, just happens ?

Examples of conflicting opinions by Economists

- inherent (Minsky, 1977)
- de-regulation of banking and investment laws
  - Yes (Ekelund and Thornton, 2008)
  - No (Calabria, 2009)

Why study financial instability ?

scientific curiosity

- what is the cause ?
- how can we measure it ?

working of a regulatory agency [Haldane and May, 2011; Berman *et al.*, 2014]

- periodically evaluates network stability
- flags[a] network *ex ante* for further analysis if its evaluation is weak

  too many false positives may drain the finite resources of the agency, but vulnerability is too important to be left for an *ex post* analysis

[a] **Flagging a network as vulnerable does not necessarily imply that such is the case, but that such a network requires further analysis based on other aspects of free market economics that cannot be modeled (*e.g.*, rumors, panic)**

Outline of talk

1. Introduction
2. **Global stability of financial system**
   - Theoretical (computational complexity and algorithmic) results
   - Empirical results (with some theoretical justifications)
   - Economic policy implications
3. Future research

UIC

To investigate financial networks, one must first settle questions of the following type:

- **What is the model of the financial network ?**
- **How exactly failures of individual financial agencies propagate through the network to other agencies ?**
- **What is an appropriate global stability measure ?**

we extend and formalize an *ex ante* graph-theoretic models for banking networks under idiosyncratic shocks

- **originally suggested by (Nier, Yang, Yorulmazer, Alentorn, 2007)**
- **directed graph with several parameters**
- **shock refers to loss of external assets**
- **network can be**
  - **homogeneous (assets distributed equally among banks)**
  - **heterogeneous (otherwise)**

Details of the model

parameterized node/edge-weighted directed graph $G = (V, E, \Gamma)$

$\Gamma = \{\mathcal{E}, \mathcal{I}, \gamma\}$

- $\mathcal{E} \in \mathbb{R}$ **total external asset**
- $\mathcal{I} \in \mathbb{R}$ **total inter-bank exposure**
- $\gamma \in (0,1)$ **ratio of equity to asset**

V is set of n banks

$\sigma_v \in [0,1]$ **weight of node $v \in V$ $\left(\sum_{v \in V} \sigma_v = 1\right)$**
share of total external asset for each bank $v \in V$

E is set of m directed edges (direct inter-bank exposures)

$w(e) = w(u,v) > 0$ **weight of directed edge $e = (u,v) \in E$**

Balance sheet details of a node (bank) v

Assets

$\iota_v = \sum_{(v,u)\in E} w(v,u)$ total interbank asset

$e_v = b_v - \iota_v + \sigma_v \mathcal{E}$ effective share of total external asset [a]

$a_v = b_v + \sigma_v \mathcal{E}$ total asset

[a] $\mathcal{E}$ is large enough such that $e_v > 0$

Liabilities

$b_v = \sum_{(u,v)\in E} w(u,v)$ total interbank borrowing

$c_v = \gamma \, a_v$ net worth (equity)

d_v customer deposit

$\ell_v = b_v + c_v + d_v$ total liability

a_v	$=$	ℓ_v	balance sheet equation
total asset		total liability	

Two banking network models

Homogeneous model

$\mathcal{E}$ and $\mathcal{I}$ are equally distributed among the nodes and edges, respectively

$$\begin{aligned} \sigma_v &= {}^{1}/_{|V|} && \text{for every node } v \in V \\ w(e) &= {}^{\mathcal{I}}/_{|E|} && \text{for every edge } e \in E \end{aligned}$$

Heterogeneous model

$\mathcal{E}$ and $\mathcal{I}$ are not necessarily equally distributed among the nodes and edges, respectively

$$\sigma_v \in (0,1) \quad \& \quad \sum_{v \in V} \sigma_v = 1$$
$$w(e) \in \mathbb{R}^+ \quad \& \quad \sum_{e \in E} w(e) = \mathcal{I}$$

UIC

How to estimate global stability ?

Via so-called "stress test"

- **give some banks a "shock"**
- **see if some of them fail**
- **see how these failures lead to failures of other banks**

Next ▶

- **how does stress ("shock") originate ?**
- **how does stress ("shock") propagate ?**

UIC

Origination of shock (initial bank failures)

Two additional parameters: $\mathcal{K}$ and Φ

- $0 < \mathcal{K} < 1$

  fraction of nodes that receive the shock

- $0 < \Phi < 1$

  severity of the shock

  i.e., by how much the external assets decrease

One additional notation: $V_{\mathbf{x}}$

$V_{\mathbf{x}}$ subset of nodes that are shocked

(how $V_{\mathbf{x}}$ is selected will be described later)

(this is the so-called "shocking mechanism")

Continued to next slide ▶

Initiation of shock of magnitude Φ

- for all nodes $v \in V_{\mathbf{x}}$, *simultaneously* decrease their external assets from e_v by $s_v = \Phi\, e_v$
  - parameter Φ determines the "severity" of the shock
- if $s_v \leq c_v$, v continues to operate with lower external asset
- if $s_v > c_v$, v *dies* (*i.e.*, stops functioning) and "propagates" shock

Next ▶
- meaning of "death" (of a node)
- how do shocks propagate ?

UIC

Global stability of financial system

How do shocks propagate ?

More notations

$\deg^{\text{in}}(v)$ = in-degree of node v

$V_{✂}(V_{\times})$ = set of dead nodes when initial shock is provided to nodes in $V_{\times}$

shocks propagate in discrete time steps $t = 1, 2, 3, \ldots$

- 1: begining initial shock
- 2: next time step

add "(t)" and "$(V_{\times})$" to indicate dependence of a variable on t and $V_{\times}$

Examples

$c_v(t, V_{\times})$: c_v at time t

$\deg^{\text{in}}(v, t, V_{\times})$: in-degree of node v at time t

$\deg^{\text{in}}$ changes because dead nodes are removed from the network

$V_{✂}(t, V_{\times})$: set of dead nodes before time t

} when initial shock is provided to nodes in $V_{\times}$

shock propagation equation

shock propagation equation

Initial shock

Big bang at $t = 1$: banking "universe" starts

$V_{✂}(1, V_{\times}) = \emptyset$ no node is dead before $t = 1$

$$c_u(1, V_{\times}) = \begin{cases} c_u - s_u, & \text{if } u \text{ was shocked } (\textit{i.e.}, \text{ if } u \in V_{\times}) \\ c_u, & \text{otherwise} \end{cases}$$

net worth of shocked nodes decrease

shock propagation equation

Initial shock

Big bang at $t = 1$: banking "universe" starts

$$V_{\text{✂}}(1, V_{\mathbf{x}}) = \emptyset$$ no node is dead before $t = 1$

$$c_u(1, V_{\mathbf{x}}) = \begin{cases} c_u - s_u, & \text{if } u \text{ was shocked (i.e., if } u \in V_{\mathbf{x}}) \\ c_u, & \text{otherwise} \end{cases}$$

net worth of shocked nodes decrease

Meaning of death of a node

If a nodes' equity becomes negative, it transmits shock and drops dead

$$\forall t_0 : c_v(t_0, V_{\mathbf{x}}) < 0 \Rightarrow v \in V_{\text{✂}}(t_0^+, V_{\mathbf{x}})$$ t_0^+ means all times after t_0

continued to next slide ▶

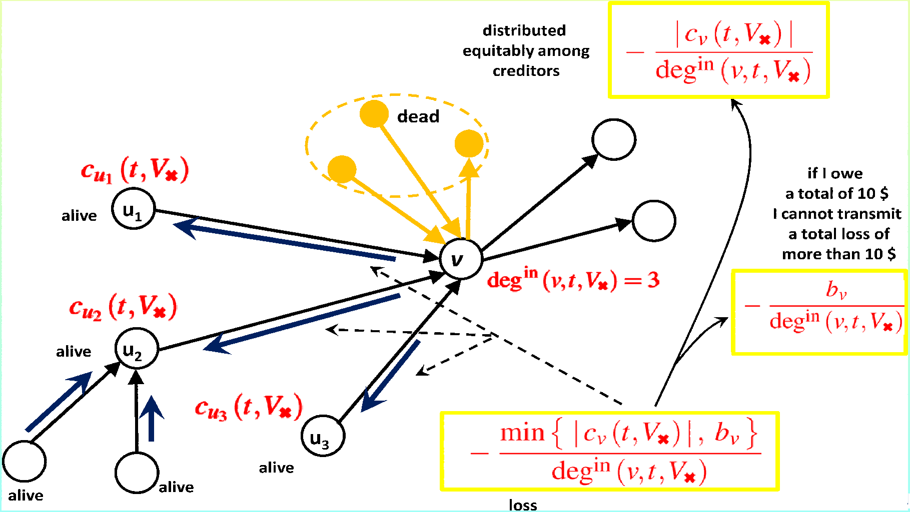
shock propagation equation (continued)

$$\forall u \in \overbrace{V \setminus V_{\text{✂}}\left(t, V_{\times}\right)}^{\text{dead nodes removed from network for all subsequent times}} :$$

$$c_u\left(t+1, V_{\times}\right) = c_u\left(t, V_{\times}\right) - \sum_{\substack{v\,:\ \left(c_v\left(t,V_{\times}\right)<0\right) \\ \wedge\left(v\in V\setminus V_{\text{✂}}\left(t,V_{\times}\right)\right) \\ \wedge\left((u,v)\in E\right)}} \frac{\min\left\{ \left|c_v\left(t, V_{\times}\right)\right| ,\ b_v \right\}}{\deg^{\text{in}}\left(v,\ t,\ V_{\times}\right)}$$

▸ Next slide: some intuition behind this equation

intuition behind individual terms of shock propagation equation



Some other models for propagation of attributes

influence maximization in social networks

[Kempe, Kleinberg,Tardos, 2003; Chen, 2008; Chen, Wang, Yang, 2009; Borodin, Filmus, Oren, 2010]

disease spreading in urban networks

[Eubank, Guclu, Kumar, Marathe, Srinivasan, Toroczkai, Wang, 2004; Coelho, Cruz, Codeo, 2008; Eubank, 2005]

percolation models in physics and mathematics

[Stauffer, Aharony, Introduction to Percolation Theory, 1994]

the model for shock propagation in banking networks is fundamentally very different from all such models

for detailed comparison, see
P. Berman, B. DasGupta, L. Kaligounder, M. Karpinski, Algorithmica (in press)

Outline of talk

1. Introduction
2. **Global stability of financial system**
   - Theoretical (computational complexity and algorithmic) results
   - Empirical results (with some theoretical justifications)
   - Economic policy implications
3. Future research

two measures of global stability

stability index of a network G

$\mathsf{SI}^*(G,T)$ — **minimum** number of nodes that need to be shocked so that all nodes in network G are dead within time T (∞ if all nodes simply cannot be put to death in any way)

$\mathsf{SI}^*(G,T) = 0.99\,\lvert V\rvert$	stability is good
$\mathsf{SI}^*(G,T) = 0.01\,\lvert V\rvert$	stability is not so good

higher $\mathsf{SI}^*(G,T)$ imply better stability

P. Berman, B. DasGupta, L. Kaligounder, M. Karpinski, Algorithmica (in press)

two measures of global stability

stability index of a network G

$\mathsf{SI}^*(G,T)$ **minimum** number of nodes that need to be shocked so that all nodes in network G are dead within time T (∞ if all nodes simply cannot be put to death in any way)

$\mathsf{SI}^*(G,T) = 0.99\,|V|$ stability is good

$\mathsf{SI}^*(G,T) = 0.01\,|V|$ stability is not so good

higher $\mathsf{SI}^*(G,T)$ imply better stability

dual stability index of a network G

$\mathsf{DSI}^*(G,T,\mathcal{K})$ **maximum** number of nodes that can be dead within time T if no more than $\mathcal{K}\,|V|$ nodes are given the initial shock

higher $\mathsf{DSI}^*(G,T)$ imply worse stability

P. Berman, B. DasGupta, L. Kaligounder, M. Karpinski, Algorithmica (in press)

two measures of global stability

stability index of a network G

$\mathsf{SI}^*(G,T)$ — **minimum** number of nodes that need to be shocked so that all nodes in network G are dead within time T (∞ if all nodes simply cannot be put to death in any way)

$\mathsf{SI}^*(G,T) = 0.99\,\lvert V\rvert$	stability is good
$\mathsf{SI}^*(G,T) = 0.01\,\lvert V\rvert$	stability is not so good

higher $\mathsf{SI}^*(G,T)$ imply better stability

dual stability index of a network G

$\mathsf{DSI}^*(G,T,\mathcal{K})$ — **maximum** number of nodes that can be dead within time T if no more than $\mathcal{K}\,\lvert V\rvert$ nodes are given the initial shock

higher $\mathsf{DSI}^*(G,T)$ imply worse stability

Two types of deaths of network G

$T = 2$	violent death!! happens too soon
$T = \infty$	slow poisoning, slow but steady

P. Berman, B. DasGupta, L. Kaligounder, M. Karpinski, Algorithmica (in press)

Global stability of financial system

Theoretical (computational complexity and algorithmic) results

some standard concepts from algorithms analysis community

approximation ratio of a maximization or minimization problem

OPT = optimal value of the objective function

ρ-approximation of a minimization problem ($\rho \geq 1$)

value of our solution $\leq \rho\,$OPT

ρ-approximation of a maximization problem ($\rho \geq 1$)

value of our solution $\geq \frac{\text{OPT}}{\rho}$

standard computational complexity classes

P, NP, $\underbrace{\text{APX-hard}}_{\text{no PTAS assuming P} \neq \text{NP}}$, $\underbrace{\text{DTIME}\left(n^{\log\log n}\right)}_{\text{quasi-polynomial time}}$ *etc*.

quasi-polynomial time: class of problems solvable in $n^{O(\log\log n)}$ time

standard classes of directed graphs

acyclic, in-arborescence *etc*.

synopsis of theoretical computational complexity results

$0 < \varepsilon < 1$ is any constant, $0 < \delta < 1$ is some constant, e is base of natural log

Network type, result type	Stability $\text{SI}^*(G,T)$ bound, assumption (if any),	Dual Stability $\text{DSI}^*(G,T,\mathcal{K})$ bound, assumption (if any)

P. Berman, B. DasGupta, L. Kaligounder, M. Karpinski, Algorithmica (in press)

synopsis of theoretical computational complexity results

$0 < \varepsilon < 1$ is any constant, $0 < \delta < 1$ is some constant, e is base of natural log

	Network type, result type	Stability $\mathsf{SI}^*(G,T)$ bound, assumption (if any),	Dual Stability $\mathsf{DSI}^*(G,T,\mathcal{K})$ bound, assumption (if any)
Homo-geneous	$T=2$ approximation hardness	$(1-\varepsilon)\ln n$, NP $\not\subseteq$ DTIME$\left(n^{\log\log n}\right)$	
	$T=2$, approximation ratio	$O\left(\log\left(\frac{\lvert V\rvert\,\Phi\,\mathcal{E}}{\gamma\,(\Phi-\gamma)\,\lvert\mathcal{E}-\Phi\rvert}\right)\right)$	
	Acyclic, $\forall\, T>1$, approximation hardness	APX-hard	$\left(1-e^{-1}+\varepsilon\right)^{-1}$, P $\neq$ NP
	In-arborescence, $\forall\, T>1$, exact solution	$O\left(n^2\right)$ time, every node fails when shocked	$O\left(n^3\right)$ time, every node fails when shocked

P. Berman, B. DasGupta, L. Kaligounder, M. Karpinski, Algorithmica (in press)

Global stability of financial system

Theoretical (computational complexity and algorithmic) results

synopsis of theoretical computational complexity results

$0 < \varepsilon < 1$ is any constant, $0 < \delta < 1$ is some constant, e is base of natural log

	Network type, result type	Stability $\mathrm{SI}^*(G,T)$ bound, assumption (if any),	Dual Stability $\mathrm{DSI}^*(G,T,\mathcal{K})$ bound, assumption (if any)
Homo-geneous	$T=2$ approximation hardness	$(1-\varepsilon)\ln n$, $\mathrm{NP} \not\subseteq \mathrm{DTIME}\left(n^{\log\log n}\right)$	
	$T=2$, approximation ratio	$O\left(\log\left(\frac{\lvert V\rvert\ \Phi\ \mathcal{E}}{\gamma\ (\Phi-\gamma)\ \lvert\mathcal{E}-\Phi\rvert}\right)\right)$	
	Acyclic, $\forall\ T>1$, approximation hardness	APX-hard	$\left(1-\mathrm{e}^{-1}+\varepsilon\right)^{-1}$, $\mathrm{P} \neq \mathrm{NP}$
	In-arborescence, $\forall\ T>1$, exact solution	$O\left(n^2\right)$ time, every node fails when shocked	$O\left(n^3\right)$ time, every node fails when shocked
Hetero-geneous	Acyclic, $\forall\ T>1$, approximation hardness	$(1-\varepsilon)\ln n$, $\mathrm{NP} \not\subseteq \mathrm{DTIME}\left(n^{\log\log n}\right)$	$\left(1-\mathrm{e}^{-1}+\varepsilon\right)^{-1}$, $\mathrm{P} \neq \mathrm{NP}$
	Acyclic, $T=2$, approximation hardness		n^{δ}, assumption ($\star$)†
	Acyclic, $\forall\ T>3$, approximation hardness	$2^{\log^{1-\varepsilon} n}$, $\mathrm{NP} \not\subseteq \mathrm{DTIME}\left(n^{\mathrm{poly}(\log n)}\right)$	
	Acyclic, $T=2$, approximation ratio‡	$O\left(\log\frac{n\ \overline{\mathcal{E}}\ \overline{w_{\max}}\ \overline{w_{\min}}\ \overline{\sigma_{\max}}}{\Phi\ \gamma\ (\Phi-\gamma)\ \underline{\mathcal{E}}\ \underline{w_{\min}}\ \underline{\sigma_{\min}}\ \underline{w_{\max}}}\right)$	

†See our paper for statement of assumption ($\star$), which is weaker than the assumption $\mathrm{P} \neq \mathrm{NP}$

‡See our paper for definitions of some parameters in the approximation ratio

---

P. Berman, B. DasGupta, L. Kaligounder, M. Karpinski, Algorithmica (in press)

UIC

brief discussion of a few proof techniques

Theorem

For homogeneous networks, SI$^*(G,2)$ cannot be approximated in polynomial time within a factor of $(1-\varepsilon)\ln n$ unless NP $\subseteq$ DTIME $\left(n^{\log\log n}\right)$

reduction from the dominating set problem for graphs

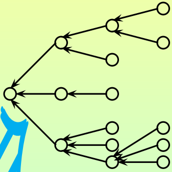
P. Berman, B. DasGupta, L. Kaligounder, M. Karpinski, Algorithmica (in press)

e.g.,

$$\deg^{in}_{max} = 3,\ \gamma = 0.1,\ \Phi = 0.15 \implies \text{SI}^*(G,T) > 0.22$$

network cannot be put to death without shocking more than 22% of the nodes

root



maximum in-degree

Theorem

For homogeneous "rooted in-arborescence" networks,

$$\text{SI}^*(G, \text{any } T) > \frac{\gamma}{\Phi \deg^{in}_{max}} \quad \text{where} \quad \deg^{in}_{max} = \max_{v \in V} \{ \deg^{in}(v) \}$$

Moreover, in this case, $\text{SI}^*(G, \text{any } T)$ can be exactly computed in $O(n^2)$ time under some mild assumption

---

P. Berman, B. DasGupta, L. Kaligounder, M. Karpinski, Algorithmica (in press)

brief discussion of a few proof techniques

Theorem

For homogeneous networks, $\mathsf{SI}^*(G,2)$ admits a polynomial-time algorithm with approximation ratio $\underbrace{O\left(\log\left(\frac{|V|\ \Phi\ \mathcal{E}}{\gamma\ (\Phi-\gamma)\ \ |\mathcal{E}-\Phi|}\right)\right)}_{\text{almost } O(\log|V|)}$

- **reformulate the problem to that of computing an optimal solution of a polynomial-size ILP**
- **use the greedy approach of [Dobson, 1982] for approximation**
- **careful calculation of the size of the coefficients of the ILP ensures the desired approximation bound**

P. Berman, B. DasGupta, L. Kaligounder, M. Karpinski, Algorithmica (in press)

Theorem (homogeneous networks, n = number of nodes)

(a) Assuming P $\neq$ NP, DSI$^*(G, \textbf{any } T, \mathcal{K})$ cannot be approximated within a factor of $\frac{1}{(1-1/e+\varepsilon)}$, for any $\varepsilon > 0$, even if G is a DAG[a]

(b) If G is a rooted in-arborescence then

$$\text{DSI}^*(G, \textbf{any } T, \mathcal{K}) < \frac{\mathcal{K}}{n}\left(1 + \deg_{\text{in}}^{\max}\left(\frac{\Phi}{\gamma} - 1\right)\right) \text{ where } \deg_{\text{in}}^{\max} = \max_{v \in V}\left\{\deg^{\text{in}}(v)\right\}$$

Moreover, in this case, DSI$^*(G, \textbf{any } T, \mathcal{K})$ can be exactly computed in $O(n^3)$ time under some mild assumption

[a] e is the base of natural logarithm

P. Berman, B. DasGupta, L. Kaligounder, M. Karpinski, Algorithmica (in press)

brief discussion of a few proof techniques

Theorem (heterogeneous networks, n = number of nodes)

Under a complexity-theoretic assumption for densest sub-hypergraph problem[a], DSI$^*(G,2,\mathcal{K})$ cannot be approximated within a ratio of $n^{1-\varepsilon}$ even if G is a DAG

[a] see B. Applebaum, Pseudorandom Generators with Long Stretch and Low locality from Random Local One-Way Functions, STOC 2012

P. Berman, B. DasGupta, L. Kaligounder, M. Karpinski, Algorithmica (in press)

Global stability of financial system

Theoretical (computational complexity and algorithmic) results

brief discussion of a few proof techniques

Theorem

For heterogeneous networks, for any constant $0 < \varepsilon < 1$, it is impossible to approximate $\text{SI}^*(G, \underbrace{\quad}_{\text{any } T>3})$ within a factor of $2^{\log^{1-\varepsilon} n}$ in polynomial time even if G is a DAG unless $\text{NP} \subseteq \text{DTIME}\left(n^{\log\log n}\right)$

- **reduction is from the MINREP problem**
  - MINREP **: a graph-theoretic abstraction of two-prover multi-round protocol for any problem in NP**
  - **Intuitively, the two provers in** MINREP **correspond to two nodes that cooperate to kill another specified set of nodes.**
- **proof is a bit technical**
  - **culminating to a set of 22 symbolic linear equations between the parameters that we must satisfy**

P. Berman, B. DasGupta, L. Kaligounder, M. Karpinski, Algorithmica (in press)

Outline of talk

1. Introduction

2. **Global stability of financial system**
   - Theoretical (computational complexity and algorithmic) results
   - Empirical results (with some theoretical justifications)
   - Economic policy implications

3. Future research

UIC

Empirical results (with some theoretical justifications)

shocking mechanism Υ: rule to select an initial subset of nodes to be shocked

Idiosyncratic shocking mechanism

[Eboli, 2004; Nier, Yang, Yorulmazer, Alentorn, 2007]
[Gai. Kapadia, 2010; May, Arinaminpathy, 2010]
[Haldane, May, 2011; Hübsch, Walther, 2012]

select a subset of $\mathcal{K}\,|V|$ nodes uniformly at random from V

can occur due to operations risks (frauds) or credit risks

Coordinated shocking mechanism

- **intuitively, nodes that are "too big to fail" in terms of their assets are shocked together**
- **belongs to the general class of *non-random correlated* shocking mechanisms**

technical details omitted from this talk

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Empirical results with some theoretical justifications

Banking network generation

why not use "real" networks ?

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Empirical results with some theoretical justifications

Banking network generation

why not use "real" networks ? several obstacles make this desirable goal impossible to achieve, *e.g.*

- **such networks with all relevant parameters are *rarely* publicly available**
- **need hundreds of thousands of large networks to have any statistical validity (in our work, we explore more than 700,000 networks)**

Empirical results with some theoretical justifications

Banking network generation

why not use "real" networks ? several obstacles make this desirable goal impossible to achieve, *e.g.*

- **such networks with all relevant parameters are *rarely* publicly available**
- **need hundreds of thousands of large networks to have any statistical validity (in our work, we explore more than 700,000 networks)**

models for simulated networks

directed scale-free (SF) model

degree distribution of nodes follow a power-law

- **defined in [Barábasi, Albert, 1999]**
- **used by prior researchers such as [Santos, Cont, 2010; Moussa, 2011; Amini, Cont, Minca, 2011; Cont, Moussa, Santos, 2010]**
- **(in our work) generated using the algorithm outlined in [Bollobas, Borgs, Chayes, Riordan, 2003]**

directed *Erdös-Rényi* (ER) model

$$\forall\, u,v \in V:\ \ \Pr\left[(u,v)\in E\right] = p$$

- **used by prior banking network researchers such as [Sachs, 2010; Gai, Kapadia, 2010; Markose, Giansante, Gatkowski, Shaghaghi, 2009; Corbo, Demange, 2010]**
- **generation algorithm is straightforward**

Empirical results with some theoretical justifications

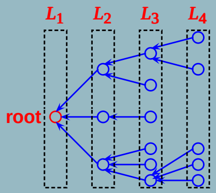
Banking network generation (continued)

we generated directed SF and directed ER networks with average degree 3 and average degree 6

In addition, we used Barábasi-Albert preferential-attachment SF model to generate in-arboescence networks

in-arborescence

- directed rooted tree with all edges oriented towards root
- belong to the class of "sparsest" connected DAG (average degree ≈ 1)
- belong to the class of "hierarchical" networks



Empirical results with some theoretical justifications

Banking network generation (continued)

For heterogeneous networks, we consider two types of inequity of distribution of assets

(0.1,0.95)-heterogeneous

- **95% of the assets and exposures involve only 10% of banks**
  - **a very small minority of banks are significantly larger than the remaining banks**

(0.2,0.60)-heterogeneous

- **60% of the assets and exposures involve only 20% of banks**
  - **less extreme situation: a somewhat larger number of moderately large banks**

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Summary of simulation environment and explored parameter space

parameter	explored values for the parameter		
network type	homogeneous		total number of parameter combinations $> 700,000$
	(α,β)-heterogeneous	$\alpha = 0.1,\ \beta = 0.95$ $\alpha = 0.2,\ \beta = 0.6$	
network topology	directed scale-free	average degree 1 (in-arborescence) average degree 3 average degree 6	
	directed Erdös-Rényi	average degree 3 average degree 6	
shocking mechanism	idiosyncratic, coordinated		
number of nodes	$50,\ 100,\ 300$		
$\mathcal{E}/\mathcal{I}$	$0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75, 2, 2.25, 2.5, 2.75, 3, 3.25, 3.5$		
Φ	$0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9$		
$\mathcal{K}$	$0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9$		
γ	$0.05,\ 0.1,\ 0.15, \ldots, \Phi - 0.05$		

To correct statistical biases, for each combination we generated 10 corresponding networks and computed the average value of the stability index over these 10 runs

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Conclusions based on empirical evaluations

Effect of unequal distribution of assets on stability

networks with all nodes having similar external assets display higher stability over similar networks with fewer nodes having disproportionately higher external assets

Conclusions based on empirical evaluations

Effect of unequal distribution of assets on stability

networks with all nodes having similar external assets display higher stability over similar networks with fewer nodes having disproportionately higher external assets

Some theoretical intuition is provided by the following lemma

Conclusions based on empirical evaluations

Effect of unequal distribution of assets on stability

networks with all nodes having similar external assets display higher stability over similar networks with fewer nodes having disproportionately higher external assets

Some theoretical intuition is provided by the following lemma

Lemma

Fix $\gamma, \Phi, \mathcal{E}, \mathcal{I}$ and the graph G. Consider any node $v \in V_{\text{✗}}$ and suppose that v fails due to the initial shock. For every edge $(u, v) \in E$, let $\Delta_{\text{homo}}(u)$ and $\Delta_{\text{hetero}}(u)$ be the amount of shock received by node u at time $t = 2$ if G is homogeneous or heterogeneous, respectively. Then,

$$\mathbb{E}\left[\Delta_{\text{hetero}}(u)\right] \geq \frac{\beta}{\alpha}\,\mathbb{E}\left[\Delta_{\text{homo}}(u)\right] = \begin{cases} 9.5\,\mathbb{E}\left[\Delta_{\text{homo}}(u)\right], & \text{if } (\alpha,\beta) = (0.1, 0.95) \\ 3\,\mathbb{E}\left[\Delta_{\text{homo}}(u)\right], & \text{if } (\alpha,\beta) = (0.2, 0.6) \end{cases}$$

This lemma implies that $\mathbb{E}[\Delta_{\text{hetero}}(u)]$ is much bigger than $\mathbb{E}[\Delta_{\text{homo}}(u)]$, and thus more nodes are likely to fail beyond $t > 1$ leading to a lower stability for heterogeneous networks

Conclusions based on empirical evaluations

Effect of unequal distribution of assets on "residual instability"

- **for homogeneous networks, if the equity to asset ratio γ is close enough to the severity of the shock Φ then the network tends to be perfectly stable, as one would intuitively expect**
- **however, the above property is *not* true for highly heterogeneous networks in the sense that, even when γ is close to Φ, these networks have a *minimum* amount of instability ("residual instability")**

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Conclusions based on empirical evaluations

Effect of unequal distribution of assets on "residual instability"

- **for homogeneous networks, if the equity to asset ratio γ is close enough to the severity of the shock Φ then the network tends to be perfectly stable, as one would intuitively expect**
- **however, the above property is *not* true for highly heterogeneous networks in the sense that, even when γ is close to Φ, these networks have a *minimum* amount of instability ("residual instability")**

to summarize

a heterogeneous network, in contrast to its corresponding homogeneous version, has a residual minimum instability even if its equity to asset ratio is very large and close to the severity of the shock

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Conclusions based on empirical evaluations

Effect of external assets on stability

$\mathcal{E}/\mathcal{I}$ controls the total (normalized) amount of external investments of all banks in the network

varying the ratio $\mathcal{E}/\mathcal{I}$ allows us to investigate the role of the magnitude of total external investments on the stability of our banking network

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Conclusions based on empirical evaluations

Effect of external assets on stability

$\mathcal{E}/\mathcal{I}$ controls the total (normalized) amount of external investments of all banks in the network

varying the ratio $\mathcal{E}/\mathcal{I}$ allows us to investigate the role of the magnitude of total external investments on the stability of our banking network

for heterogeneous banking networks, global stabilities are affected very little by the amount of the total external asset in the system

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Conclusions based on empirical evaluations

Effect of network connectivity (average degree) on stability

prior observations by Economists

- networks with **less** connectivity are **more** prone to contagion [Allen and Gale, 2000]

  rationale: more interbank links may also provide banks with a type of co-insurance against fluctuating liquidity flows

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Conclusions based on empirical evaluations

Effect of network connectivity (average degree) on stability

prior observations by Economists

- **networks with less connectivity are more prone to contagion [Allen and Gale, 2000]**

  rationale: more interbank links may also provide banks with a type of co-insurance against fluctuating liquidity flows

- **networks with more connectivity are more prone to contagion [Gai and Kapadia, 2008]**

  rationale: more interbank links increases the opportunity for spreading insolvencies to other banks

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Conclusions based on empirical evaluations

Effect of network connectivity (average degree) on stability

prior observations by Economists

- networks with **less** connectivity are **more** prone to contagion [Allen and Gale, 2000]

  rationale: more interbank links may also provide banks with a type of co-insurance against fluctuating liquidity flows

- networks with **more** connectivity are **more** prone to contagion [Gai and Kapadia, 2008]

  rationale: more interbank links increases the opportunity for spreading insolvencies to other banks

Actually, both observations are correct **depending on the type of network**

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Global stability of financial system

Empirical results (with some theoretical justifications)

Conclusions based on empirical evaluations

Effect of network connectivity (average degree) on stability

homogeneous network

higher connectivity leads to lower stability

heterogeneous network

higher connectivity leads to higher stability

our paper provides theoretical insights behind these observations

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Conclusions based on empirical evaluations

phase transitions of properties of random structures are often seen

Example (**giant component formation in Erdos-Renyi random graphs** $\forall u, v \in V: \ \Pr\left[(u,v) \in E\right] = p$)

$p \leq {}^{(1-\varepsilon)}/_{n}$ $\Rightarrow$ **with high probability all connected components have size** $O(\log n)$

$p \geq {}^{(1+\varepsilon)}/_{n}$ $\Rightarrow$ **with high probability at least one connected component has size** $\Omega(n)$

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Conclusions based on empirical evaluations

phase transitions of properties of random structures are often seen

Example (**giant component formation in Erdos-Renyi random graphs** $\forall u,v \in V: \ \Pr\left[(u,v) \in E\right] = p$)

$p \leq {}^{(1-\varepsilon)}/_{n}$ $\Rightarrow$ **with high probability all connected components have size** $O(\log n)$

$p \geq {}^{(1+\varepsilon)}/_{n}$ $\Rightarrow$ **with high probability at least one connected component has size** $\Omega(n)$

phase transition properties of stability

- **denser** ER **and** SF **networks, for smaller value of** $\mathcal{K}$, **show a sharp decrease of stability when** γ **was decreased beyond a particular threshold**
- **homogeneous in-arborescence networks under coordinated shocks exhibited a sharp increase in stability as** ${}^{\mathcal{E}}/_{\mathcal{I}}$ **is increased beyond a particular threshold provided** $\gamma \approx {}^{\Phi}/_{2}$

our paper provides theoretical insights behind this observation

---

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Software

interactive software FIN-STAB implementing shock propagation algorithm available from `www2.cs.uic.edu/~dasgupta/financial-simulator-files`

Outline of talk

1. Introduction
2. **Global stability of financial system**
   - Theoretical (computational complexity and algorithmic) results
   - Empirical results (with some theoretical justifications)
   - Economic policy implications
3. Future research

Global stability of financial system

Economic policy implications

when to flag the financial network for potential vulnerabilities ?

- **equity to asset ratios of most banks are low,**

  or,

- **the network has a highly skewed distribution of external assets and inter-bank exposures among its banks and the network is sufficiently sparse,**

  or,

- **the network does not have either a highly skewed distribution of external assets and a highly skewed distribution of inter-bank exposures among its banks, but the network is sufficiently dense**

B. DasGupta, L. Kaligounder, Journal of Complex Networks 2(3), 313-354 (2014)

Outline of talk

1. Introduction

2. Global stability of financial system
   - Theoretical (computational complexity and algorithmic) results
   - Empirical results (with some theoretical justifications)
   - Economic policy implications

3. **Future research**

UIC

Future research

Future research questions

Our results are only a first step towards understanding vulnerabilities of banking systems

Further investigate and refine the network model

- network topology and parameter issues
  - network structures that closely resembles "real" banking networks
  - optimal networks structures for a stable financial system

Effect of "diversified" external investments on the stability

Other notions of stability

- percentage of the external assets that remains in the system at the end of shock propagation

Questions with policy implications

- identifications of modifications of network topologies or parameters to turn a vulnerable system to a stable one

Final slide

Thank you for your attention

Questions??